

Ime in priimek

Osnove matematične analize: 3. računski izpit

7. september 2021

Time limit is 60 minutes. You may use 2 A4-sized sheets of paper with formulas. The use of electronic devices (calculator, phone) is prohibited. Justify all your answers!

Write each problem onto a separate sheet of paper. If you are writing onto a blank piece of paper rather than the problem sheet, please sign your name at the top of every page, write the problem number at the top as well and scan the problems in the correct order. Thanks!

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Vpisna številka

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1. naloga (30 točk)

Using induction show that 13 divides $3^{4n-1} + 4^{2n+1}$ for all integers $n \geq 1$.

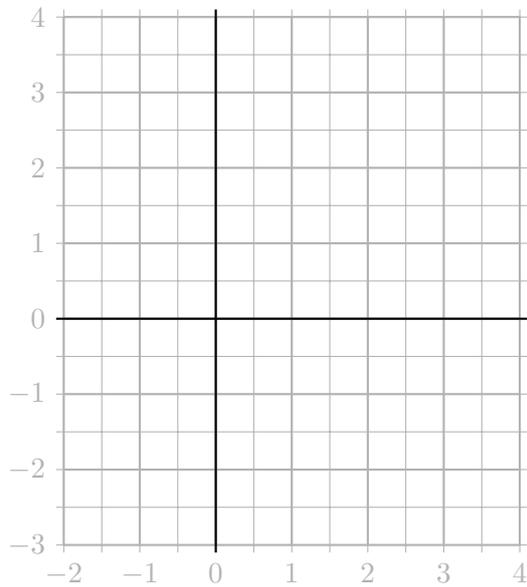
2. naloga (35 točk)

The function f is defined by

$$f(x) = \begin{cases} \frac{x^2-1}{a(x+1)}, & x \leq -1, \\ \cos(\pi x) + b, & -1 < x \leq 2, \\ \frac{x-2}{\ln(x-2)}, & x > 2. \end{cases}$$

a) (15 točk) Determine a and b so that f is continuous at $x = -1$ and at $x = 2$.

b) (15 točk) For these a and b sketch the graph of f (determine the zeros and the poles as well as limits at $\pm\infty$; you do not need to compute the derivatives).



c) (5 točk) Is f injective?

3. naloga (35 točk)

a) (10 točk) Compute the indefinite integral of

$$f(x) = x \left(x - \frac{\pi}{2} \right).$$

b) (15 točk) Compute the indefinite integral of

$$g(x) = \sin(x) \cos^3(x).$$

c) (10 točk) Compute the area bounded by the graphs of f and g .

Hint: The graphs of f and g intersect only where they are both zero.