Computational Topology

Exam 1, June 20, 2017

You have 60 minutes to solve the problems.

This is an open book exam. You are allowed to use books and notes. You are not allowed to use calculators, ipads, phones or neighbors. Any form of communication is forbidden and will result in a negative grade in all past and current work in this class.

1. a For students of RI: Let $A = \{(x, y) : |x| + |y| \leq 1\} \subset \mathbb{R}^2$. Plot the set A in the plane. Is A
   - convex?
   - open?
   - closed?
   - homeomorohic to the unit circle?
   - conractible?

   Provide arguments for your answers!

   b For student of IŠRM2: Prove: Every simplicial complex K is the geometric realization of the nerve of the family of its vertex stars.

2. A simplicial complex K contains the following simplices $\{v0, v1, v2, v3, v4, \langle v0, v1\rangle, \langle v1, v2, v3\rangle, \langle v0, v2\rangle,$
   - Add the smallest possible number of missing simplices (if any are missing).
   - Write down the star $\mathrm{St}(v_0)$ and the link $\mathrm{Lk}(v_0)$ of the vertex $v0$.
   - Write down the matrices for the boundary operators $\partial_2 C_2 \to C_1$ and $\partial_1 C_1 \to C_0$
   - Write down a set of generators for the cycle group Z_1 and the boundary group B_1
   - What are the Betti numbers of K?

3. On the complex K from the previous problem
   - construct a discrete gradient vector field with 2 critical vertices and 3 critical edges; how many ctirical triangles will it have?
   - associate values of an injective discrete Morse function, corresponding to your gradient vector field,
   - construct the sublevel complex filtration and plot the peristence diagram.

4. Compute the bottleneck distance between the followibg two persistence diagrams